Sirindhorn International Institute of Technology

Thammasat University at Rangsit

School of Information, Computer and Communication Technology

TCS 455: Problem Set 6 Solution

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Course Title: Mobile Communications

Instructor: Dr. Prapun Suksompong (prapun@siit.tu.ac.th)

Course Web Site: http://www.siit.tu.ac.th/prapun/ecs455/

1. Recall that the baseband OFDM modulated signal can be expressed as

$$s(t) = \sum_{k=0}^{N-1} S_k \frac{1}{\sqrt{N}} 1_{[0,T_s]}(t) \exp\left(j\frac{2\pi kt}{T_s} \right)$$

where $S_0, S_1, \ldots, S_{N-1}$ are the (potentially complex-valued) messages.

Let $T_s = 1$ [ms], *N* = 8, and

$$(S_0, S_1, \ldots, S_{N-1}) = (1-j, 1+j, 1, 1-j, -1-j, 1, 1-j, -1+j)$$

Solution

a.

b. $\text{Re}\{s(t)\} = a(t) - b(t)$

2. Consider the discrete-time complex FIR channel model

$$y[n] = \{h * x\}[n] + w[n] = \sum_{m=0}^{2} h[m] x[n-m] + w[n]$$

where $w[n]$ is zero-mean additive Gaussian noise.

In this question, assume that *h*[*n*] has unit energy and that $H(z)$ has two zeros at

$z_1 = \rho e^{j\frac{2\pi}{3}}$ and $z_2 = \frac{1}{\rho}$ where $\rho < 1$.

Solution

The channel h has two zeros at $z_1 = \rho e^{j\frac{2\pi}{3}}$ and $z_2 = \frac{1}{\rho}$.

Thus, before normalized to unit energy, we have

$$H_{un}(z) = \left(1 - z_1 z^{-1}\right)\left(1 - z_2 z^{-1}\right) = 1 - \left(z_1 + z_2\right) z^{-1} + z_1 z_2 z^{-2}.$$

Inverse Z-transform gives

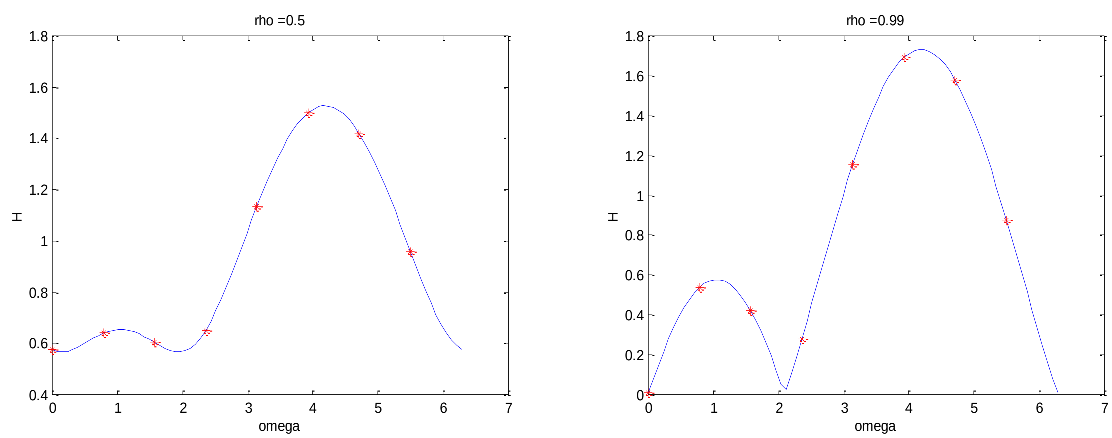
$$h_{un}[k] = \delta[k] - \left(z_1 + z_2\right)\delta[k-1] + z_1 z_2 \delta[k-2].$$

The energy of $h_{un}[k]$ is $E_{h_{un}} = 1^2 + \left|z_1 + z_2\right|^2 + \left|z_1\right|^2 \left|z_2\right|^2$.

So,

$$h[k] = \frac{1}{\sqrt{E_{h_{un}}}} h_{un}[k] \text{ and } H(z) = \frac{1}{\sqrt{E_{h_{un}}}} H_{un}(z).$$

a. The plots of $\left|H\left(e^{j\omega}\right)\right| = \left|H(z)\big|_{z=e^{j\omega}}\right|$ in the range $0 \le \omega \le 2\pi$ for ρ = 0.5 and 0.99 are shown below:



b. For OFDM system with block size N = 8, find the corresponding channel gains $H_k = H(z)\big|_{z=e^{j\frac{2\pi}{N}k}}$, k = 0, 1, 2, …, N-1 for ρ = 0.5 and 0.99.

Ch # k	$\rho = 0.5$		$\rho = 0.99$	
	$H\left(e^{j\frac{2\pi}{N}k}\right)$	$\left\lvert H\left(e^{j\frac{2\pi}{N}k}\right)\right\rvert$	$H\left(e^{j\frac{2\pi}{N}k}\right)$	$\left\lvert H\left(e^{j\frac{2\pi}{N}k}\right)\right\rvert$
0	-0.5455 + 0.1890i	0.5774	-0.0087 + 0.0050i	0.0100
1	0.1407 + 0.6246i	0.6403	0.5170 + 0.1489i	0.5380
2	0.4657 + 0.3858i	0.6047	0.3710 - 0.2026i	0.4227
3	0.4649 + 0.4555i	0.6508	-0.0624 + 0.2716i	0.2787
4	0.9820 + 0.5669i	1.1339	0.5860 + 0.9949i	1.1547
5	1.4881 - 0.1882i	1.4999	1.6375 + 0.4284i	1.6927
6	0.8436 - 1.1417i	1.4196	1.3609 - 0.7973i	1.5773
7	-0.3480 - 0.8919i	0.9574	0.2171 - 0.8489i	0.8762

3.

4. In this question, the channel noise is non-zero. $w[n]$ is now i.i.d. complex-valued Gaussian noise. Its real and imaginary parts are i.i.d. zero-mean Gaussian with variance $N_0/2$ where

$$N_0 = \frac{2}{3\mathrm{SNR}_{\mathrm{norm}}}.$$

Assume $\mathrm{SNR}_{\mathrm{norm}}$ is 2 dB.

Solution

a. Complete the following table.

Ch # k	$\rho = 0.5$	$\rho = 0.99$
0	0.5490	0.7399
1	0.5207	0.5657
2	0.5407	0.6076
3	0.5227	0.6516

4	0.3412	0.3348
5	0.2316	0.1782
6	0.2565	0.2068
7	0.4095	0.4383

Remark: The SER of ch#0 when H_0 is about 0 should be very close to 0.75. This is because the channel gain destroys almost all the information contained in the received signal. Hence, the ML detector will be correct with probability 0.5 for each dimension. The complex number (QPSK symbol) has two dimensions. Hence, the chance that it will be decoded correctly is $0.5 \times 0.5 = 0.25$.